

*An Account of the several Species of Infinite Quantity,
and of the Proportions they bear one to the other,
as it was read before the Royal Society, by
E. Halley.*

THAT all Magnitudes infinitely great, or such as exceed any assignable Quantity, are equal among themselves, though it be vulgarly received for a Maxim, is not yet so common as it is erroneous; and the reason of the Mistake seems to be, That the Mind of Man, coming to contemplate the Extensions of what exceeds the bounds of its Capacity, and of which the very Idea does include a negation of Limits; it comes to pass that we acquiesce generally, and it suffices to say such a Quantity is infinite.

But if we come more nearly to examine this Notion, we shall find, that there are really besides infinite *Length* and infinite *Area*, no less than Three several sorts of infinite Solidity: all of which are *Quantitates sui generis*, having no more relation or proportion the one to the other, than a Line to a Plane, or a Plane to a Solid, or a Finite to an Infinite: but that, among themselves, each of those Species of Infinites are in given Proportions, is what I now intend to make plain, if possible.

But first, infinite *Length* or a Line infinitely long is to be considered either as beginning at a point, and so infinitely extended one way, or else both ways from the same Point; in which case the one, which is a beginning Infinity, is the one half of the whole, which is the sum of the beginning and ceasing Infinity, or as I may say of Infinity *a parte ante* and *a parte post*, which is analogous to Eternity in time or Duration, in which there is always as much to follow as is past from any point.

point or moment of Time: Nor doth the Addition or Subduction of finite Length or Space of time alter the case either in Infinity or Eternity, since both the one or the other cannot be any part of the whole.

As to Infinite *Surface* or *Area*, any right Line, infinitely extended both ways on an infinite Plane, does divide that infinite Plane into equal parts, the one to the right and the other to the left of the said Line; but if from any point in such a Plane two right Lines be infinitely extended so as to make an Angle, the infinite Area, intercepted between those infinite right Lines, is to the whole infinite Plane, as the Arch of a Circle, on the point of concurrence of those Lines as a Centre, intercepted between the said Lines is to the Circumference of the Circle; or as the Degrees of the Angle to the 360 Degrees of a Circle. For example, two right Lines meeting at a right Angle do include, on an infinite Plane, a quarter part of the whole infinite Area of such a Plane.

But if so be two parallel infinite Lines be supposed drawn on such an infinite Plane, the Area intercepted between them will be likewise infinite; but at the same time will be infinitely less than that Space which is intercepted between two infinite Lines that are inclined, tho' with never so small an Angle, for that in the one case the given finite Distance of the parallel Lines diminishes the Infinity in one degree of Dimension; whereas in a Sector, there is Infinity in both Dimensions: and consequently the Quantities are the one infinitely greater than the other, and there is no Proportion between them.

From the same Consideration arise the Three several Species of infinite Space or Solidity, as has been said, for a Parallelepiped or a Cylinder infinitely long is greater than any finite Magnitude how great soever, and all such Solids supposed to be formed on given Bases,

are as those Bases, in proportion to one another. But if two of these Three Dimensions are wanting, as in the space intercepted between two parallel Planes infinitely extended and at a finite distance; or with infinite Length and Breadth with a finite Thickness: all such Solids shall be as the given finite Distances one to another; but these Quantities, tho' infinitely greater than the other, are yet infinitely less than any of those wherein all the three Dimensions are infinite. Such are the Spaces intercepted between two inclined Planes infinitely extended; the Space intercepted by the Surface of a Cone or the sides of a Pyramid likewise infinitely continued, &c. of all which notwithstanding, the Proportions one to another, and to the $\pi \pi \pi$ or vast Abyss, of infinite space (wherein is the *Locus* of all things that are or can be; or to the solid of infinite Length, Breadth and Thickness taken all manner of ways) are easily assignable. For the space between two Planes is to the whole, as the Angle of those Planes to the 360 Degrees of the Circle. As for Cones and Pyramids they are as the spherical Surface, intercepted by them, is to the Surface of the Sphere, and therefore Cones are as the Verted sines of half their Angles, to the Diameter of the Circle: These three sorts of infinite Quantity are analogous to a Line, a Surface and Solid, and after the same manner cannot be compared, or have no proportion the one to the other

Besides these, there are several other Species of infinite Quantity, arising from the contemplation of Curves and their Asymptotes, which by reason of the difficulty of the subject cannot be made so plain to most Readers; but what has been already said may be sufficient to evince what we undertook to explain.